

Workshop on Gray products and lax constructions

Raitenhaslach 2025

February 5, 2025

Background Talks

GRAY'S ORIGINAL TENSOR PRODUCT (*1 talk*)

The Gray tensor product was first introduced in the setting of strict 2-categories, where it is far more amenable to direct computation. This talk will survey the original definition and construction from [Gra74] and its first properties — in particular, the connection to lax transformations. Time permitting, the talk may also cover homotopical aspects of Gray's construction (e.g., the Quillen bifactoriality of [Lac04]) or variants (such as the variant representing pseudonaturality of [GPS95]).

References

- [Gra74] John W. Gray, *Formal category theory: adjointness for 2-categories*, Lecture Notes in Mathematics, Vol. 391. Springer-Verlag, Berlin-New York, 1974.
- [GPS95] Robert Gordon, John Power, Ross Street. *Coherence for tricategories*, Mem. Amer. Math. Soc. 117 (1995), no. 558
- [Lac04] Stephen Lack, *A Quillen model structure for bicategories*, K-theory, 33(3) (2004)

The $(\infty, 2)$ -categorical theory

THE SCALED SIMPLICIAL SET APPROACH (*1 talk*)

This talk should briefly introduce scaled simplicial sets, and the construction of their Gray product, following §2 of [GHL2]. It should then survey the relation to functor categories with lax (or oplax) transformations, and, time permitting, describe the universal property in terms of oplax normal functors.

THE MODEL-INDEPENDENT APPROACH (1 talk)

The second talk of this series should survey the results of [CM23] providing a model-independent characterization of the Gray product of $(\infty, 2)$ -categories

in terms of cubes. In particular, the talk should cover the universal properties of Corollary 3.5 in *op. cit.*, which characterize the Gray product in terms of its operation on cubes.

[GHL21] Andrea Gagna, Yonatan Harpaz, and Edoardo Lanari. *Gray tensor products and Lax functors of $(\infty, 2)$ -categories*. Advances in Mathematics, Volume 391, 2021

[CM23] Timothy Champion and Yuki Maehara. *A model-independent Gray tensor product for $(\infty, 2)$ -categories*. arXiv:2304.05965

The (∞, ∞) - and ω -categorical theories

THE GRAY TENSOR PRODUCT OF (∞, n) -CATEGORIES (1 talk)

This talk should introduce cubical sets and marked cubical sets, and discuss the way in which they are expected to model (∞, ∞) -categories. The speaker should then cover the construction of the Gray tensor products of marked cubical sets in §2.2 of [CKM20]. Time permitting, the talk can also discuss some of the model-categorical aspects of the Gray product of marked cubical sets.

THE GRAY TENSOR PRODUCT OF (STEINER) ω -CATEGORIES (2 talks)

Talk 1: This talk will survey the theory of Steiner ω -categories and their relation to augmented directed chain complexes, following the background in §1 of [AGOR23], supplemented by the original reference [Ste04].

Talk 2: Using the definitions and machinery established in the previous talk, this talk should introduce the tensor products of augmented directed complexes and strong Steiner ω -categories (§7 of [Ste04]). It should discuss the monoidal closed-ness of this product, and, time permitting, its relation to the Gray product constructed in [AS93].

[CKM20] Tim Champion, Chris Kapulkin, and Yuki Maehara. *A cubical model for (∞, n) -categories*. arXiv:2005.07603

[AGOR23] Dimitri Ara, Andrea Gagna, Viktoriya Ozornova, Martina Rovelli. *A categorical characterization of strong Steiner ω -categories*. Journal of Pure and Applied Algebra, vol. 227, no. 7, 2023

[Ste04] Richard Steiner. *Omega-categories and chain complexes*. Homology, Homotopy and Applications, vol 6(1), 2004

[AS93] Fahd Ali Al-Agl, Richard Steiner, *Nerves of Multiple Categories*, Proceedings of the London Mathematical Society, Volume s3-66, Issue 1, January 1993

Minicourses

A three-talk series by Fernando Abellán, and a three-talk sequence by Hadrian Heine. Talks to be announced.